Numerical Modeling of Turbulent Zero Momentum Late Wakes in Density Stratified Fluids

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Abstract

Direct numerical simulations were performed of late wakes, with and without net momentum, in unstratified and stably density stratified fluids. Where data was available, comparisons were made with tank experiments to verify simulation results. After judging the simulations to adequately represent real flows, further analysis was performed on the simulation results to understand the flow in more detail than was previously possible. Since the simulation data provide a fully three-dimensional (3D) description of the flow, detailed, precise and fully 3D analysis is possible, providing answers to questions not directly addressable by considering 2D slices of experimental data. The topology of vortex lines was visualized, and the relative contributions of various sources of vorticity were computed, revealing how vortex stretching, advection, buoyancy and viscosity affect the evolution of the flow. We conclude that the numerical approach used in this study provides a valid and useful complement to analytical and experimental studies, and provides significant insights into the explanation of 3D phenomena which previously eluded researchers of stratified turbulent wakes.

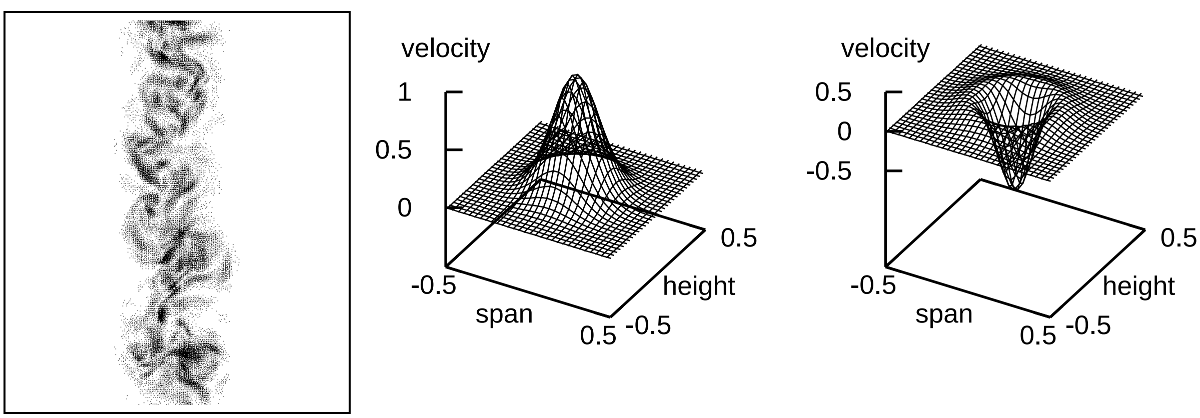
1 Introduction

Turbulent late wakes have been studied for decades using analytical techniques and experiments. The earliest analytical results (e.g. Townsend 1956, 1976; Tennekes & Lumley 1972) hypothesize that transport of mean streamwise momentum by streamwise mean flow balances cross-stream transport of turbulent momentum, and this reasoning leads to a quantitative description of asymptotic behavior at late times of self-similar solutions for flows in unstratified fluids. Although details of the original hypothesis have met with some contention, experiments verified the predicted scaling laws and asymptotic behavior (e.g., Bevilaqua & Lykoudis 1978) for wakes with a non-zero momentum deficit. Wakes in fluids with stable density stratification display significant qualitative and quantitative differences from unstratified flows; in stratified flows, the near wake first expands as in the unstratified case, then collapses vertically (Schooley & Stewart 1962), then coherent horizontal "pancake" vortices appear (Pao & Kao 1977, Lin & Pao 1979, Hopfinger 1987) and the late time asymptotic behavior for non-zero momentum wakes, while arguably described as self-similar, exhibits a scaling law which has not yet been adequately explained (Lin *et al.* 1992; Chomaz, Bonneton & Hopfinger 1993; Chomaz *et al.* 1993; Spedding, Browand & Fincham 1996a, 1996b).

The previously performed wake experiments lack the ability to reveal sufficient data about the three-dimensional (3D) structure of flows, but numerical simulations can complement tank experiments by providing precise and copious 3D data, assuming the simulations adequately model real flows. The research described in this paper aims to provide such a numerical model. This study includes wakes with and without net momentum deficit, with and without stratification. Initially turbulent flows with zero momentum model wakes behind submerged non-accelerating self-propelled bodies. Initially turbulent flows with non-zero momentum model wakes behind submerged towed or accelerating bodies. This paper focuses on results for wakes with zero momentum deficit. We refer the interested reader to Gourlay *et al.* (2000) for results concerning wakes with non-zero momentum.

Section 2 describes the code, initial conditions and simulation parameters. In section 3, comparisons with analytical predictions and tank experiments provide confidence in the viability of the simulations, and we extract some novel results from fully 3D analyses, including results cogent to the problem of surface signatures of the wakes. We conclude in section 4 that this model provides a valuable complement to other research and we indicate future directions of study and other questions this model can answer about turbulent wakes.

2 Simulation procedures



"turbulence" momentum deficit profile zero momentum profile

Figure 1: Components of simulation initial conditions: turbulence and streamwise velocity profiles. The turbulence is windowed with a cylindrical Gaussian. The initial wake width is used as the characteristic length scale of the problem, so all spatial units are in terms of the initial wake width, defined as 1.0.

The direct numerical simulation software used in this study (Julien *et al.* 1996) codifies the Boussinesq approximation of the fluid dynamics equations using a spectral, uniform spatial representation with periodic boundary conditions. A half-wave Fourier basis represents vertical functions, modeling stress-free boundaries a the top and bottom. A third order

Runge-Kutta integration method (Spalart, Moser & Rogers 1991) advances the solution in time.

Initial conditions include wakes with and without momentum deficit, and with and without density stratification. (Due to length constraints this paper focuses on wakes with zero momentum.) Two components constitute the initial velocity field: a mean streamwise velocity profile and spectrally specified "turbulence". The mean profiles included a Gaussian, $exp(-r^2/(2\rho^2))$, for the wake with momentum, an ad-hoc function, $exp(-r^2/(2\rho^2))*(r^2/\rho^2-1.0)$ for the zero-momentum wake, and no mean profile to test properties of the initial "turbulence" seeding in the absence of mean shear. Figure 1 shows these components.

The initial Reynolds numbers, $Re_0 = U_0 L_0/\nu$ for the numerical experiments discussed here are 5000 and 10000, where U_0 is the initial mean profile streamwise velocity amplitude, $L_0 = \rho$ is the initial wake width, and ν is the kinematic viscosity of the fluid.

3 Simulation results and analysis

Ideally, analytical predictions would produce results which would provide for a quantitative comparison to verify the numerical model, but the prevailing analytical model fails to represent real flows with zero momentum. That analysis of the time-averaged momentum equations (e.g. Townsend 1956, 1976; Tennekes & Lumley 1972) assumes a self-similar behavior of the flow, but such self-similarity does not occur for zero-momentum flows, as figure 2 clearly demonstrates. Tennekes & Lumley (1972) suggest reasons why that analysis might fail for zero-momentum wakes; in the derivation, an indefensible assumption is made: that the Reynolds stress is constant.

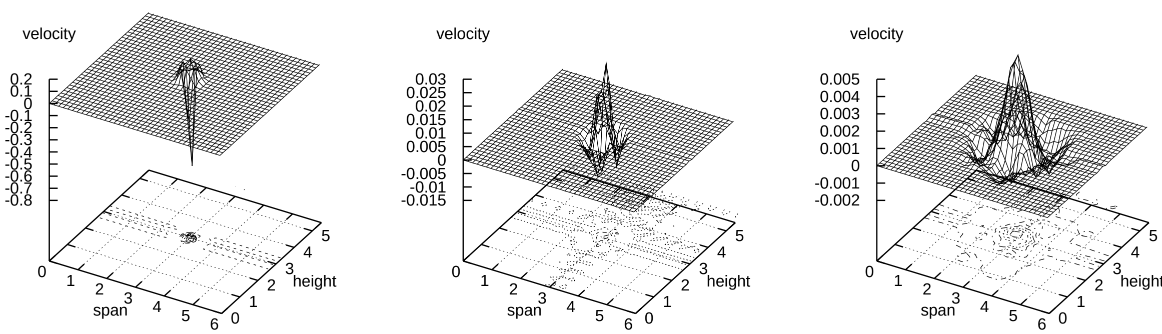


Figure 2: Mean streamwise velocity profile evolution for Re_0=10000, unstratified. Times are 0.0, 3.37 and 10.3 in units of L_0/U_0.

Although the mean velocity profile rapidly decays, it provides kinetic energy for conversion into turbulence, and therefore differs from a wake with no mean profile. The rapid decay and lack of self-similarity could imply that the detailed form of the initial wake profile perhaps matters little in the manifestation of the late wake, except in non-generic details. If so, then the viability of the simulation results does not depend on the particular form of the mean velocity profile.

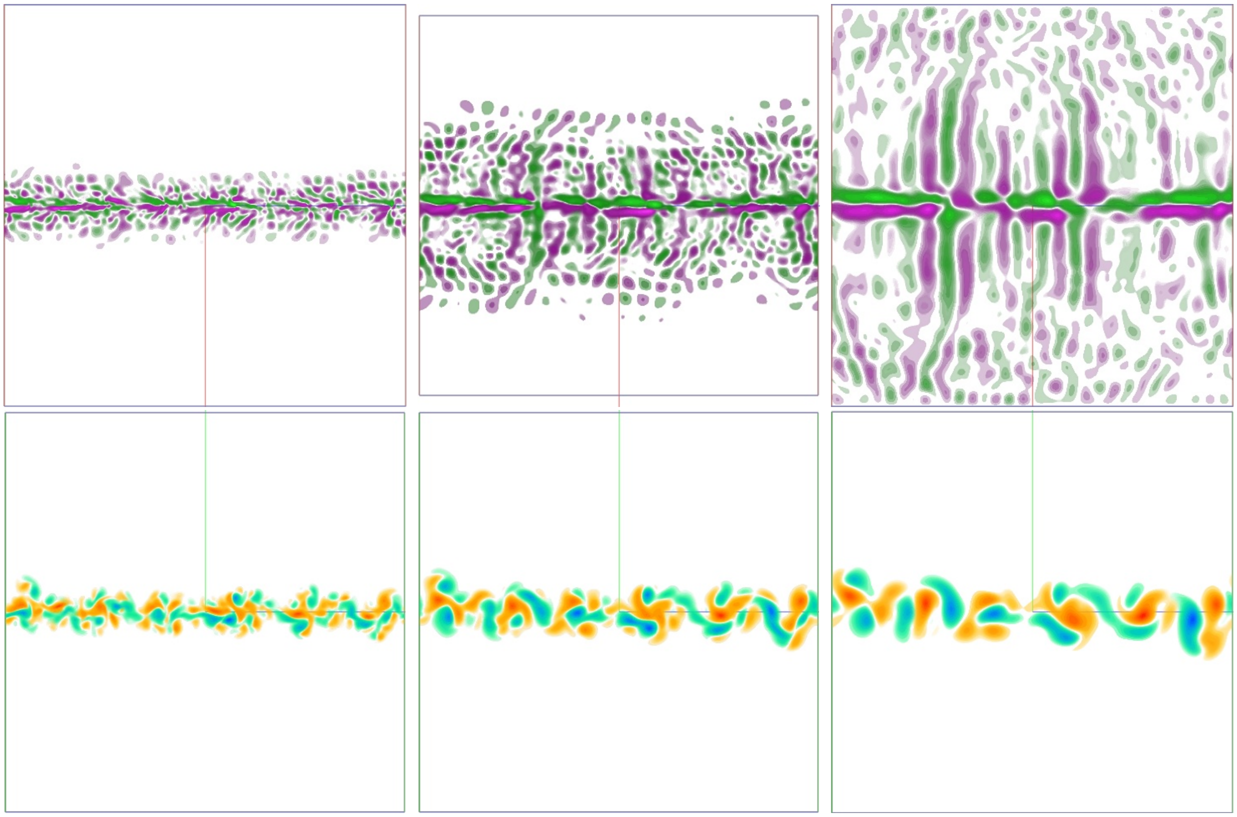


Figure 3: Vorticity evolution for Re_0=5000, Fr_0=5 simulation with zero-momentum (self-propelled) initial mean wake profile. Upper three images show vertical slices of spanwise vorticity, where green and magenta indicate opposite signs. Lower three images show horizontal slices of vertical vorticity where blue and orange indicate opposite signs. Times, in units of Nt, from left to right, are 19.3, 53.3 and 141.7.

Tank experiments by Schooley & Stewart (1962), Lin & Pao (1979), and Alan Brandt (private communication) provide a basis for comparison and verification of the numerical model. The major qualitative behaviors for stratified wakes include three stages of evolution: initial conical expansion, vertical collapse with internal wave generation, and horizontal spreading with coherent vortices and less vertical spreading. Lin & Pao also indicate that the late wake coherent vortices behind self-propelled bodies form earlier and with smaller length scale than for towed bodies. Figure 3 shows components of vorticity on plane slices through the 3D domain for the Re_0=5000, Fr_0=5 simulation with zero-momentum (self-propelled) initial mean velocity profile. The vertical vorticity images show horizontal wake spreading. The horizontal vorticity images show a narrow, vertically limited, band of concentrated vorticity, along with a rapidly propagating, baroclinically generated field of horizontal vorticity. Aside from the baroclinically generated field, the simulation manifests a wake which spreads horizontally but not vertically. At late times, the wake field contains several coherent counter-rotating vortex pairs. Vortices with the same sign pair to form larger vortices, and by this process the vorticity field length scale increased in steps. Figure 4 shows the vortex topology of the late wake vortices, revealing that the vortex pairs, previously described as horizontal vortices and called "pancake vortices", are actually vortex rings.

The rapid vertical propagation of horizontal vorticity, suggests a method for how the submerged wake communicates a signal to the surface. The vertical propagation of horizontal vorticity shown in figure 3 is confined in the spanwise direction to a region as wide as the wake. The vertical vorticity field lacks this propagation because baroclinicity produces only horizontal vorticity. Unstratified wakes also lack vertical propagation of horizontal vorticity in the same configuration. Vertically propagating vorticity has not been observed in towed body wakes, but could have been overlooked due to smaller relative magnitude. Visualization of baroclinic production of vorticity reveals concidence of horizontal baroclinicity with the observed vertically propagating horizontal vorticity.

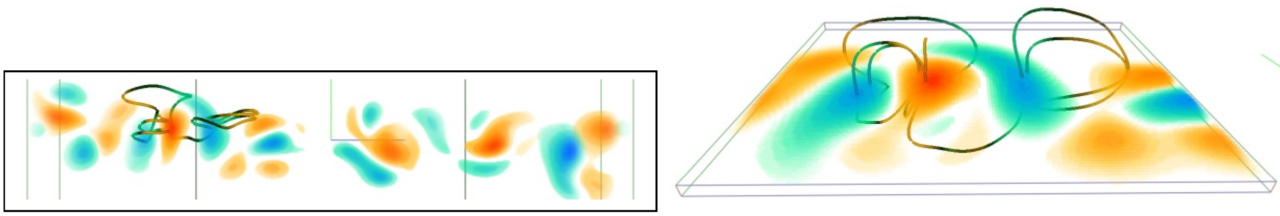


Figure 4: Vorticity topology of late wake coherent vortices for Re_0=5000, Fr_0=5 simulation with zero-momentum at time 141.7 Nt. Left image shows whole domain from above. Right image shows close-up from oblique perspective. Blue and orange clouds show vertical vorticity, and lines show vortex integral lines.

4 Conclusions

At this early stage of our research, the direct numerical simulation succeeds primarily in demonstrating its ability to adequately model real wake flows, both for unstratified and stratified fluids. This success opens opportunities for highly detailed and fully 3D analysis of the wake flow and vorticity field. Already the simulations have revealed the topology of vorticity, and shown vertical propagation of horizontal vorticity which suggests a candidate method for how the wake transmits a signature to the fluid surface.

We intend to learn how the coherent late wake vortices remain for long durations. This question relates to the question of how stratified wakes (whose length scales increase in time) differ from isotropic homogenous turbulent flow (whose length scales decrease in time). A detailed look at the balance of vortex generation terms will provide some clues. We expect to find a balance between baroclinic production and viscous dissipation at late times. Presumably that balance maintains coherency in the late wake vortices.

Other open areas of practical interest include the effects of shear and variable stratification on late wakes, especially of late wakes with zero net momentum (i.e. wakes of self-propelled bodies). Study of those effects might suggest ways to mitigate the surface signature.

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